# Math 109 Midterm 1 Practice

**Print Name:** _____ **Section:** _____

**Statement of Ethics regarding this exam**

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

**Signature:**_____ **Date:** _____

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- This is a 50 minute **closed book** exam. **No notes, books, or calculators are allowed.**
- Present your solution to each problem in a clear and orderly fashion. **Show all your work.** An answer without justification will not receive full credit.
- Do not use any techniques we have not covered in class yet.
- This exam contains 7 pages (including this cover page) and 6 questions. The last page is intended for use as scrap paper.

The table on the right is for grading purposes. Please do not write in it.

| Question | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 10 | |
| 6 | 10 | |
| Total: | 100 | |

1. (20 points) Compute the following integral

$$\int_1^3 x^2 \ln(2x) dx$$



**Points earned: _____ out of 20 points**

2. (20 points) Compute the following integral

$$\int \sec^4 x \tan^2 x dx$$



**Points earned: _____ out of 20 points**

3. (20 points) Compute the following integral

$$\int \frac{x^2}{\sqrt{4-x^2}} dx$$



**Points earned: _____ out of 20 points**

4. (20 points) Compute the following integral

$$\int \frac{3x^2 + 4x + 2}{x^3 + 2x^2 + 2x} dx$$



**Points earned: _____ out of 20 points**

5. (10 points) Does the following improper integral converge? If so, find its value.

$$\int_1^\infty \frac{\ln x}{x^2} dx$$



**Points earned: _____ out of 10 points**

6. (10 points) Suppose a function $f(x) \neq 0$ satisfies the differential equation $y' = (x^2-4)\cdot y$. Find the interval on which the function $g(x) = \ln|f(x)|$ is decreasing. (You do not need to solve the differential equation to answer this question!)



**Points earned: _____ out of 10 points**

*These pages are intended for use as scrap paper.*

# Trigonometric Identities

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

- Sum and Difference Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

- Double Angle Formulas

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

- Half Angle Formulas

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

- Product Formulas

$$\sin A \sin B = \frac{1}{2}\left(\cos(A - B) - \cos(A + B)\right)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A - B) + \cos(A + B)\right)$$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A + B) + \sin(A - B)\right)$$

# Differentiation Formulas

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{x^2 + 1}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

# Integration Formulas

$$\int \frac{1}{x}dx = \ln|x| + C$$

$$\int a^x dx = \frac{1}{\ln a}a^x + C$$

$$\int \ln x dx = x\ln x - x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \cot x dx = -\ln|\csc x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}}dx = \frac{1}{a}\sec^{-1}\frac{|x|}{a} + C$$

# Trigonometric Substitution

$$\sqrt{a^2 - x^2} \Longrightarrow x = a\sin\theta$$

$$\sqrt{a^2 + x^2} \Longrightarrow x = a\tan\theta$$

$$\sqrt{x^2 - a^2} \Longrightarrow x = a\sec\theta$$